Measuring Transformer Coupling Factor k

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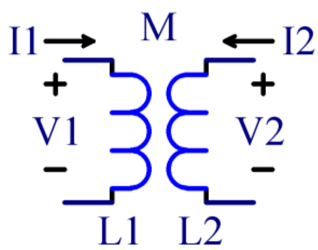

A transformer with individual winding inductances L_1 and L_2 has mutual inductance M between the windings. Transformer terminal equations are:

$$V_1 = j\omega(L_1 \cdot I_1 + M \cdot I_2) \quad (1)$$
$$V_2 = j\omega(M \cdot I_1 + L_2 \cdot I_2) \quad (2)$$

If winding 2 is shorted, V_2 becomes zero so equation (2) becomes:

$$0 = j\omega(M \cdot I_1 + L_2 \cdot I_2)$$

Solving for I_2:

$$I_2 = \frac{-M \cdot I_1}{L_2} \quad (3)$$

Substitute equation (3) into equation (1):

$$V_1 = j\omega(L_1 \cdot I_1 - \frac{M^2 \cdot I_1}{L_2}) = I_1 \cdot j\omega(L_1 - \frac{M^2}{L_2}) \quad (4)$$

If you define

$$L_S = L_1 - \frac{M^2}{L_2} \quad (5)$$

then equation (4) is in the form

$$V_1 = I_1 \cdot j\omega L_s$$

which is simply the voltage and current relationship of an inductor. L_s is therefore the inductance measured across L_1 with winding 2 shorted. Solving equation (5) for M^2:

$$M^2 = L_2(L_1 - L_s) \quad (6)$$

The definition of transformer coupling factor k is

$$k = \frac{M}{\sqrt{(L_1 \cdot L_2)}}$$

or

$$k^2 = \frac{M^2}{L_1 \cdot L_2} \qquad (7)$$

Substitute equation (6) into equation (7):

$$k^2 = \frac{L_2(L_1 - L_s)}{L_1 \cdot L_2} = \frac{(L_1 - L_s)}{L_1} = 1 - \frac{L_s}{L_1}$$

or

$$\boxed{k = \sqrt{(1 - \frac{L_s}{L_1})}} \qquad (8)$$

L_1 is the inductance measured across L_1 with winding 2 open and L_s is the same measurement with winding 2 shorted. K is determined by inserting these inductance values into equation (8).