Coordinates of the Reciprocal of a Vector

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Given a vector *V* directed from the origin to the point *(x, y)*, what are the coordinates of $\frac{1}{V}$? By definition, the inverse of a vector with magnitude M and angle θ , written as $M \measuredangle \theta$, is $\frac{1}{M} \measuredangle -\theta$. We have $V \equiv (x, y) = M \measuredangle \tan^{-1} \frac{y}{x}$ where $M = \sqrt{x^2 + y^2}$, so

$$\frac{1}{V} = \frac{1}{M} \measuredangle -\tan^{-1} \frac{y}{x}$$

$$= \left(\frac{1}{M} \cos\left(\tan^{-1} \frac{y}{x} \right), \frac{1}{M} \sin\left(\tan^{-1} \frac{y}{x} \right) \right)$$ ordered pair of x, y coordinates

$$= \left(\frac{1}{M} \cdot \frac{1}{\sqrt{1 + \left(\frac{y}{x} \right)^2}}, \frac{1}{M} \cdot \frac{\left(\frac{y}{x} \right)^2}{\sqrt{1 + \left(\frac{y}{x} \right)^2}} \right)$$ from trig identities

$$= \left(\frac{1}{M} \cdot \frac{x}{\sqrt{x^2 + y^2}}, \frac{1}{M} \cdot \frac{-y}{\sqrt{x^2 + y^2}} \right)$$

$$= \left(\frac{x}{M^2}, \frac{-y}{M^2} \right) = \left(\frac{x}{x^2 + y^2}, \frac{-y}{x^2 + y^2} \right)$$

Either of the final results is quite simple, especially if M is known beforehand.