Calculate Inductor AC Flux Density

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Peak flux density in a power inductor is required to predict core losses and to avoid saturation. You can calculate it from either the voltage across the inductor or the current through it. This paper addresses AC sinusoidal waveforms.

A circumflex ^ over a variable below means the peak value of a sinusoid. In all formulas, numerical results will be correct if you stay in a given system of units (like SI).

To calculate peak flux from the inductor voltage start with Faraday's law

$$v = -N \cdot \frac{d\,\Phi}{dt}$$

which states that the voltage across an inductor equals the turns times the rate of change of flux which links the turns. The negative sign indicates that the voltage opposes the flux polarity.

For a sinusoid of varying flux

$$\begin{aligned} v &= -N \frac{d}{dt}\left(\hat{\Phi}\cos(\omega t)\right) \\ &= -N\,\hat{\Phi}(-\omega)\sin(\omega t) \\ &= 2\pi f N \hat{\Phi}\sin(\omega t) \end{aligned}$$

and the peak value is therefore

$$\begin{aligned} \hat{v} &= 2\pi f N \hat{\Phi} \\ &= 2\pi f N \hat{B} A \end{aligned}$$

where $\hat{B}$ is flux density and A is cross-sectional area of the core. Solving for $\hat{B}$:

$$\boxed{\hat{B} = \frac{\hat{v}}{2\pi f N A}} \qquad (1)$$

This is a familiar formula for flux density in terms of applied voltage. It is also seen with v_{RMS} in the numerator and $\sqrt{2}$ instead of 2 in the denominator.

Now to find peak flux density dependency on peak current $\hat{i}$. Since $v = i Z$ by Ohm's law or $\hat{v} = \hat{i}(2\pi f L)$ for an inductor we can substitute into (1) with the result:

$$\hat{B} = \frac{\hat{i}\,L}{N\,A} \qquad (2)$$

If the inductance is known this formula may be used directly. Note that the flux density no longer depends on *f* in this form.

Starting from the relationship between flux density and magnetic field strength we can get another form which shows the dependency of peak flux density on permeability and magnetic path length:

$$\hat{B} = \mu\hat{H} = \mu\frac{N\,\hat{i}}{l} \qquad (3)$$

Permeability here is expressed as relative permeability times the permeability constant $\mu = \mu_r \mu_0 = \mu_r \cdot 4\pi \cdot 10^{-7}\,H/m$. The magnetic path length here is magnified by the presence of a gap in the magnetic path: $l = l_m + \mu l_g$ where l_m is the path length in the magnetic material and l_g is the path length in the gap.

Note that (3) looks very different from (1). If a voltage is applied across an inductor as in (1), you *increase N* to decrease *B*, but if the inductor current is fixed as in (3) you *reduce N* to decrease *B*. Also, you increase *cross-sectional area* to decrease *B* when a voltage is applied, but you increase *path length* when the current is fixed.

To determine which formula to use, consider what contributes to flux in the core. When the inductance is the primary of a transformer (1) should be used (this includes current transformers[1]). The only flux in the core of a transformer is due to the magnetizing current arising from the primary voltage. Additional primary current due to the load does not cause additional core flux.

In other situations, such as using the inductor in the series leg of a lowpass filter, it may be more appropriate to use (2) or (3). In this case the current through the inductor is known, it is the only contributor to core flux, and the voltage across it is small.

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1 A current transformer is actually a transformer, so only a small part of the primary current (the magnetizing current) contributes to core flux. The majority of the primary current is transformed to the load by the turns ratio. (1) should be used to calculate flux density by first calculating the (usually tiny) voltage across the primary, which is normally only a single turn. It is not correct to calculate $\hat{B}$ from (2) or (3) using the primary current for $\hat{i}$. Those formulas would only apply if the secondary were open-circuited.